Numerical experiments with an interior-exterior point method for Semidefinite Programming

DerkaouiOrkia†, and Ahmed Lehireche*
† University Dr Moulay Tahar of Saida, Computer Science Department, BP 138 ENNASR City Saida 20000, Algeria, E-mail: orkya1@yahoo.fr
*University DjillaliLiabes of SidiBel Abbes, Computer Science Department, BP 89 City Sidi Bel Abbes 22000, Algeria, E-mail: elhir@univ-sba.dz

Abstract

The SemiDefinite Programming (SDP) is not only an extension of Linear Programming (LP) but also includes convex quadratic optimization problems and some other convex optimization problems. It has a lot of applications in various fields such as combinatorial optimization [29], control theory [28], robust optimization [27, 30] and quantum chemistry [25, 26]. In our work we present the implantation of a new algorithm for solving semidefinite programming problems. The algorithm is based on a new class of interior–exterior method [22, 31]. The latter is also known as the primal-dual method of type path-following where only one Newton iteration is sufficient to approximate the solution of penalized problem which satisfies a criterion of proximity.

The result is demonstrated by solving problems from SDPLIB problem sets [24] using semidefinite solver SDPA [23] that is modified to include the interior-exterior point method subroutine. We specifically solved instances of quadratic problems.

The preliminaries numerical results show the performance of this procedure and why its integration is a reasonable approach for solving semidefinite programming problems. Our future work is to implement a new variant of this method with another way to determine the step-size along the direction which is more efficient than classical line searches.

Keywords. Interior point method, Exterior point method, Primal-dual method, Semidefinite programming, Interior–exterior approach.

References

[1] A. Auslender, R. Cominetti and M. Haddou. 1997. “Asymptotic analysis of penalty and barrier methods in convex and linear programming.” Math.Oper. Res., vol. 22, pp. 43–62.
[2] A. Auslender, M. Tebboule and S. Ben-Tiba. 1999. “Interior proximal and multipliers methods based on second-order homogeneous kernels.” Math.Oper. Res. , vol. 24, no. 3, pp. 645–668.
[3] A. Ben-Tal and M. Zibulevsky. 1997. “Penalty-barrier methods for convex programming problems.” SIAM J. Optim., vol. 7, pp. 347–366.
[4] A. Ben-Tal, B. Yuzefovich and M. Zibulevsky. 1992. “Penalty-barrier multipliers methods for minimax and constrained smooth convex optimization.”Optimization Laboratory, Technion, Israel. Research Report 9-92.
[5] G. Debreu. 1952. “Definite and Semidefinite Quadratic Forms”, Economet-rica, vol. 20, pp. 295–300.
[6] A. Fiacco, G. McCormick. 1990. “Nonlinear Programming Sequential Un-constrained Minimization Technique.” SIAM Classic Appl. Math. SIAM Philadelphia, PA.
[7] I. Griva. 2004. “Numerical experiments with an interior-exterior point method for Nonlinear Programming”. Computational Optimization and Applications, vol. 29, no. 2, pp. 173–195.
[8] Yu Nesterov and A. Nemirovski. 1994. “Interior-Point Polynomial Algorithms in Convex Programming”, SIAM, Philadelphia.

[9] Yu Nesterov and B. Polyak. 2006. Cubic regularization of Newton method and its global performance, Math. Programming, vol. 108, no. 1, pp. 177–205.
[10] B. Polyak. 1987. Introduction to Optimization. Software Inc.: NY.
[11] R. Polyak. 1992. “Modified barrier functions,” Math. Programming, vol.54, pp. 177–222.
[12] R. Polyak. 1988. “Smooth optimization methods for minmax problems, SIAM Journal on Control and Optimization, vol. 26, no. 6, pp. 1274–1286.
[13] R. Polyak. 2001. “Log-sigmoid multipliers method in constrained optimization.” Ann. Oper. Res., vol. 101, pp. 427–460.
[14] R. Polyak and M. Teboulle. 1997. “Nonlinear rescaling and proximal-like methods in convex programming,” Math. Programming, vol. 76, pp. 265–284.
[15] R. Polyak. 2002. “Nonlinear rescaling vs. smoothing technique in convex optimization,” Math. Programming, vol. 92, pp. 197–235.
[16] R. Polyak and I. Griva. 2004. “Primal-dual nonlinear rescaling methods for convex optimization,” JOTA, vol. 122, no. 1, pp. 111–156.
[17] I. Griva and R. Polyak. 2006. “Primal-dual nonlinear rescaling method with dynamic scaling parameter update,” Math. Programming, vol. 106, pp. 237–259.
[18] R. Polyak. 2006. “Nonlinear rescaling as interior quadratic prox method for convex optimization,” Computational Optimization and Applications, vol.35, no. 3, pp. 347–373.
[19] D. F. Shanno and R. J. Vanderbei. 1999. “An interior point algorithm for nonconvex nonlinear programming,” COAP, vol. 13, pp. 231–252.
[20] P. Tseng and D. Bertsekas. 1993. “On the convergence of the exponential multipliers method for convex programming,” Math. Programming, vol. 60, pp. 1–19.
[21] H.YAMASHITA AND T.TANABE. 2010. “A primal-dual exterior point method for nonlinear optimization, ” SIAM J. OPTIM.Vol. 20, No. 6, pp. 3335–3363
[22] K. EL YASSINI. 2000. “ Approche de type intérieur-extérieur pour la programmation linéaire et la programmation quadratique convexe ,” PhD Thesis., University of Sherbrooke, Canada.
[23] Fujisawa, K. and Kojima, M. SDPA (Semidefinite Programming Algorithm) Users Manual. Technical Report b-308, Tokyo Institute of Technology, (1995).
[24] B. Borchers, SDPLIB 1.2, a library of semidefinte programming test problems, Optimization Methods and Software 11 & 12 (1999) 683-690.
[25] M. Nakata, H. Nakatsuji, M. Ehara, M. Fukuda, K. Nakata and K. Fujisawa, Variational calculations of fermion second-order deduced density matrices by semidefinite programming algorithm, Journal of Chemical Physics 114 (2001) 8282-8292.
[26] M. Nakata, M. Ehara and H. Nakatsuji, Density matrix variational theory: Application to the potential energy surfaces and strongly correlated systems, Journal of Chemical Physics 116 (2002) 5432-5439.
[27] A. Ben-Tal and A. Nemirovskii, Lectures on Moden Convex Optimizatin Analysis, Alogorithms, and Engineering Applications, (SIAM, Philadelphia, 2001).
[28] S. Boyd, L. E. Ghaoui, E. Feron and V. Balakrishnan, Linear matrix inequalities in system and control theory, Society for Industrial and Applied Mathematics 1994 Philadelphia, PA, ISBN 0-89871-334-X.
[29] M. X. Goemans and D. P. Williamson, Improved approximation alogrithoms for maxmum cut and satisfiability problems using semidefinite programming, Journal of Association for Computing Machinery 42(6) (1995) 1115-1145.
[30] H. Wolkowicz, R. Saigal and L. Vandenberghe, Handbook of Semidefinite Programming, Theory, Algorithms, and Applications, (Kluwer Academic Publishers, Massachusetts, 2000).
[31] K. EL YASSINI. “ An interior-exterior approach for convex quadratic programming”, Journal Applied Numerical Mathematics Volume 62 Issue 9, September, (2012) Pages 1139-1155